

Written Exam at the Department of Economics Winter 2017–18

## **Advanced International Trade**

3-hour closed-book exam

February 13 2018

### **SUGGESTED ANSWERS**

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This document consists of 6 pages in total.

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## Problem 1:

Consider the Dornbusch, Fischer and Samuelson (1977) model with two countries, Home and Foreign. Each country produces a continuum of goods indexed  $z \in (0, 1)$ . The only factor of production is labor which is paid the wage  $w$  in Home and  $w^*$  in Foreign. The labor endowments of Home and Foreign are given by  $L$  and  $L^*$ . Consumers have identical CES preferences. The utility function of a representative Home consumer is:

$$U = \left[ \int_0^1 c(z)^{(\sigma-1)/\sigma} dz \right]^{\sigma/(\sigma-1)} \quad (1)$$

where  $c(z)$  is the consumption level of good  $z$  and  $\sigma > 1$  is the constant elasticity of substitution. The consumer's budget constraint is:

$$w = \int_0^1 p(z)c(z)dz \quad (2)$$

where  $p(z)$  is the price of good  $z$ . Let wages in Home be the numeraire, i.e.,  $w = 1$ .

1. Show that the consumer spends the following share of the budget on good  $z$ :

$$s(z) = \frac{p(z)c(z)}{w} = \frac{p(z)^{1-\sigma}}{\int_0^1 p(z)^{1-\sigma} dz} \quad (3)$$

Provide a short and complete explanation of the relationship in (3)

*Suggested answer:*

Households maximize utility subject to their budget constraint. The Lagrangian is:

$$L = \left[ \int_0^1 c(z)^{(\sigma-1)/\sigma} dz \right]^{\sigma/(\sigma-1)} + \lambda \left( w - \int_0^1 p(z)c(z)dz \right)$$

FOCs are:

$$\frac{\partial L}{\partial c(z)} = \left[ \int_0^1 c(z)^{(\sigma-1)/\sigma} dz \right]^{1/(\sigma-1)} c(z)^{-1/\sigma} - p(z) = 0$$

Taking ratios of FOCs for two varieties ( $z$  and  $z'$ ):

$$\frac{c(z)^{-1/\sigma}}{c(z')^{-1/\sigma}} = \frac{p(z)}{p(z')} \Leftrightarrow c(z) = c(z') \left( \frac{p(z)}{p(z')} \right)^{-\sigma}$$

Insert the above expression in the budget constraint:

$$w = \int_0^1 p(z)c(z') \left( \frac{p(z)}{p(z')} \right)^{-\sigma} dz \Leftrightarrow c(z') = \frac{p(z')^{-\sigma}}{\int_0^1 p(z)^{1-\sigma} dz} w$$

Budget shares are then calculated as:

$$s(z) = \frac{p(z)c(z)}{w} = \frac{p(z)^{1-\sigma}}{\int_0^1 p(z)^{1-\sigma} dz} = \left( \frac{p(z)}{P} \right)^{1-\sigma}$$

where  $P = \left( \int_0^1 p(z)^{1-\sigma} dz \right)^{\frac{1}{1-\sigma}}$  is the CES price index. Budget shares are a function of a good's price relative to the overall price index of the economy: A higher relative price of good  $z$  lowers the budget share spent on  $z$  since  $\sigma > 1$ . While budget shares may not be uniform across goods, they add up to one, i.e.,  $\int_0^1 s(z)dz = 1$ .

2. In the free trade equilibrium, countries produce the goods in which they have a comparative advantage. Suppose Home produces all goods in  $(0, z')$ , while Foreign produces all goods in  $(z', 1)$ , where  $z'$  is the good for which production costs are exactly the same in the two countries. This condition is one of two equilibrium conditions in the Ricardian trade model of Dornbusch, Fischer and Samuelson (1977). Define and describe using (3) the second condition they use to determine the equilibrium values for  $z'$  and the relative wages,  $w/w^*$ .

*Suggested answer:*

The second equilibrium condition is the balanced trade condition, dictating that Home's exports are equal to Home's imports:

$$\theta(z')w^*L^* = (1 - \theta(z'))wL$$

where  $\theta(z')$  is the fraction of income spent on goods produced in Home. Using (3), this is defined as:

$$\theta(z') = \frac{\int_0^{z'} p(z)^{1-\sigma} dz}{\int_0^1 p(z)^{1-\sigma} dz}$$

This gives the second equilibrium condition:

$$\frac{w}{w^*} = \frac{\theta(z')}{1 - \theta(z')} \frac{L^*}{L}$$

which is identical to the balanced trade condition in the original article except for the definition of  $\theta(z')$ .

3. Consumer welfare is equal to  $w/P$ , where  $P$  is the aggregate CES price index. Does welfare increase when the two countries are allowed to trade with each other? If so, describe the sources of gains from trade. What happens to the consumption of goods when transitioning from autarky to free trade? Are the changes in consumption different from Dornbusch, Fischer and Samuelson (1977) who assume Cobb-Douglas utility?

*Suggested answer:*

Since welfare is equal to  $1/P$ , there are gains from trade if the autarky price index,  $P^A$ , is greater than the free trade price index,  $P^F$ . After opening up to trade, Home produces  $z \in [0, z']$  whose prices are the same before and after opening up to trade since Home wages are the numeraire. The remaining goods  $z \in (z', 1)$  are now imported by Home consumers since they are produced at lower costs in Foreign. The prices of these imported goods will, in other words, be lower relative to their autarky prices. For that reason, the overall price index decreases when the two countries are allowed to trade with each other and this is why there are gains from trade. Nominal income of Home consumers are fixed, but the lower free trade price index implies that Home consumers spent a greater share of their income on goods produced in Home (even though their prices are unchanged). In other words, Home consumers' real income increases and so does their consumption of goods produced in Home. Since the overall budget share spent on goods  $z \in [0, z']$  increases, the overall budget share spent on goods  $z \in [z', 1]$  must go down despite the lower prices

of these goods. Without knowing how the prices of goods  $z \in [z', 1]$  change between autarky and free trade, one can only conclude that the budget share for at least one imported good decreases. With Cobb-Douglas utility, Home consumers also experience an increase in their real income due to the lower prices of the imported goods. In this case, they spent a fixed share of their income on each good and trade allows them to increase their real consumption of the goods  $z \in [z', 1]$ , while leaving their consumption of goods produced in Home unchanged. Consumption responses are therefore quite different for CES and Cobb-Douglas utility specifications.

## Problem 2:

Consider an economy with two industries. Industry 1 is a perfectly competitive industry, producing a homogeneous good  $y$  using labor and capital. Let  $c_1(w, r)$  denote the unit cost of industry 1, where  $w$  denotes wages and  $r$  capital rents. Industry 2 is a monopolistically competitive industry, with each firm producing a unique variety of a differentiated good using labor and capital. Let  $c_2(w, r)$  denote the marginal cost of industry 2 and assume fixed costs to equal  $\alpha c_2(w, r)$ . That is, the fixed costs use labor and capital in the same proportions as the marginal costs. Consumers spend  $\mu \in [0, 1]$  on the good produced by industry 1 and  $1 - \mu$  on the varieties produced by industry 2. Demand for the differentiated goods are:

$$x = \left(\frac{p}{P}\right)^{-\sigma} \frac{(1 - \mu)I}{P} \quad (4)$$

where  $\sigma > 1$  is the constant elasticity of substitution parameter,  $p$  the price of a given variety,  $P$  the price index for the differentiated goods and  $I$  income. Let the price of the homogeneous good be the numeraire.

1. Write down the relationship between the prices of goods and factor prices. Does the Stolper-Samuelson theorem hold for this economy?

*Suggested answer:*

Prices are optimally set by equating marginal revenue and marginal costs:

$$\begin{aligned} 1 &= c_1(w, r) \\ p &= c_2(w, r) \frac{\sigma}{\sigma - 1} \end{aligned}$$

where  $\sigma/(\sigma - 1)$  is the standard CES markup and  $c_j(w, r)$  is the unit cost function for industry  $j = 1, 2$ . Totally differentiating these equations lead to:

$$\begin{aligned} 0 &= \theta_{L1} \hat{w} + (1 - \theta_{L1}) \hat{r} \\ \hat{p} &= \theta_{L2} \hat{w} + (1 - \theta_{L2}) \hat{r} \end{aligned}$$

where  $\theta_{Lj}$  is the cost share of labor in industry  $j$ . These equations relate changes in goods prices to changes in factor prices. Note that the constant markup disappears when considering price changes instead of levels. As such, we have the usual two equations in two unknowns ( $w$  and  $r$ ) that are used to derive the Stolper-Samuleson theorem for the  $2 \times 2$  Heckscher-Ohlin model. And for that reason, the theorem

also holds in the case with one industry being monopolistically competitive. The Stolper-Samuleson theorem states that a relative price increase of a good will lead to a rise in the return to the factor used most intensively in the production of the good, and conversely, to a fall in the return to the other good.

2. Show that the output of each firm in industry 2 is given by:

$$x = (\sigma - 1)\alpha \quad (5)$$

Provide a short and complete account of the relationship in (5).

*Suggested answer:*

Firms earn zero profits in the monopolistically competitive industry. Combining this with the monopolistic pricing rule lead to:

$$\begin{aligned} \pi &= 0 \\ &= px - c_2(w, r)(\alpha + x) \\ &= \frac{\sigma}{\sigma - 1}c_2(w, r)x - c_2(w, r)(\alpha + x) \\ \implies \frac{1}{\sigma - 1}x &= \alpha \\ x &= (\sigma - 1)\alpha \end{aligned}$$

Output increases with a higher degree of product substitutability and higher fixed costs (relative to marginal cost).

3. Write down the full-employment conditions for the two factors. Does the Rybczynski theorem hold for this economy?

*Suggested answer:*

The full-employment conditions are:

$$\begin{aligned} L &= a_{L1}y + a_{L2}(\alpha + x)n \\ K &= a_{K1}y + a_{K2}(\alpha + x)n \end{aligned}$$

where  $a_{fj}$  is the unit demand for factor  $f$  when producing good  $j$  and  $y$  the total output of industry 1. There are  $n$  firms in industry 2 using labor and capital for their fixed and variable costs of production. Each firm produces an output level of  $x$  with  $\alpha$  being the additional output lost to the fixed costs. Since  $x$  is fixed by exogenous parameters ( $\sigma$  and  $\alpha$ ), this gives us two equation in two unknowns,  $y$  and  $n$ . As such, we have the same setup that is used to derive the Rybczynski theorem for the  $2 \times 2$  Heckscher-Ohlin model, except industry 2's total output changes because the number of firms changes in response to factor endowment changes. As such, the Rybczynski theorem holds in this case as well. That is, a rise in the endowment of one factor will lead to a more than proportional expansion of the output in the sector which uses that factor intensively, and an absolute decline of the output of the other good.

### Problem 3:

Answer True or False to each of the statements below. Briefly explain your answer.

1. In the monopolistic competition model of Krugman (1979), firms differ in terms of their productivity and only the most productive firms export to international markets.

*Suggested answer:*

False. Firms are equally productive in Krugman (1979) and they all export. Melitz (2003) uses productivity differences and the presence of variable and fixed trade costs to rationalize why only the most productive firms export.

2. An economy transitioning from restricted trade to free trade will always generate welfare gains for everyone.

*Suggested answer:*

False. Samuelson (1939) argues that it is possible to construct a feasible transfer system that makes households no worse off going from autarky to free trade. Without this policy intervention, trade is likely to lead to welfare losses for some households.

3. The Armington model predicts that larger countries export larger quantities of each good.

*Suggested answer:*

True. The Armington model assumes that each country produces one unique good. Larger countries will therefore export larger quantities of their good relative to smaller countries. Hummels and Klenow (2005) show that the intensive margin explains around 40 percent of the greater exports of larger economies.

4. The empirical study of Hummels, Jørgensen, Munch and Xiang (2014) show that offshoring increases the wages of low-skilled workers due to the productivity effect.

*Suggested answer:*

False. Hummels et al. (2014) find that offshoring decreases the wages of low-skilled workers which is consistent with the displacement effect being stronger than the productivity effect.